Math Virtual Learning

Precalculus with Trigonometry

May 21, 2020

Precalculus with Trigonometry
Lesson: May 21st, 2020

Objective/Learning Target:
Students will Learn how to determine the area of triangles when they can’t use A = ½bh.

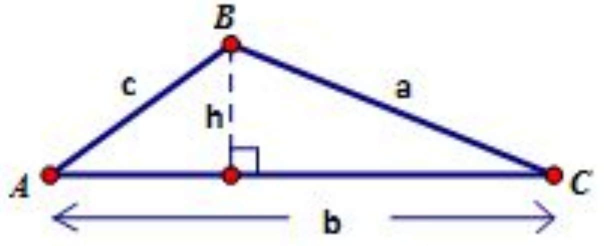
Let’s Get Started:

As early as elementary school you learned that you could determine the area of a triangle by using the formula: $A = \frac{1}{2}bh$. This formula requires you to know a side which we call the base and the perpendicular distance from that side to the opposite angle. Unfortunately we don’t always know that information. In this lesson you will learn a couple other techniques for finding the area of a triangle. The following video shows how to use one of these techniques.

Watch Video: How to Find the Area of a Triangle Using Trigonometry

Technique 1 - Area of Oblique Triangles

So technique 1 uses trig to alter the familiar formula for area to a formula that doesn't require the height as long as we know an angle and the two sides adjacent to that angle. In other words, if you have SAS information, you can use the formulas below.



$$Area = \frac{1}{2}bc(\sin A)$$

$$Area = \frac{1}{2}ab(\sin C)$$

$$Area = \frac{1}{2}ac(\sin B)$$

<u>Technique 2 - Heron’s Formula</u>

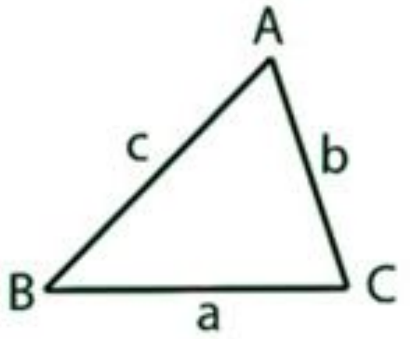
The following technique is approximately 2000 years old and is attributed to Heron of Alexandria. This technique can be used in non-right triangles where you know all of the sides, but none of the angles. Watch the following video to see how to use Heron’s Formula

Watch Video: [Heron’s Formula - Khan Academy]

Heron’s Formula

$$s = \frac{a+b+c}{2}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$



Example #1:

Find the area of a triangular lot having two sides of lengths 90 meters and 52 meters and an included angle of 102°.

Solution

Consider $a = 90$ meters, $b = 52$ meters, and angle $C = 102°$, as shown in Figure 6.8. Then, the area of the triangle is

$$\text{Area} = \frac{1}{2}ab \sin C = \frac{1}{2}(90)(52)(\sin 102°) \approx 2289 \text{ square meters.}$$

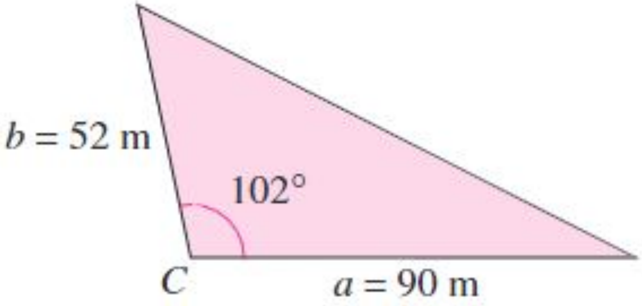


FIGURE 6.8

Example #2:

Find the area of a triangle having sides of lengths $a = 43$ meters, $b = 53$ meters, and $c = 72$ meters.

Solution

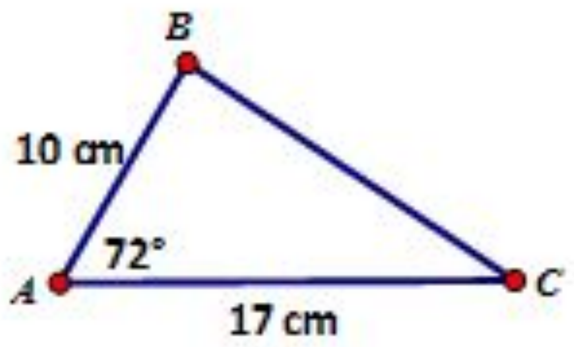
Because $s = (a + b + c)/2 = 168/2 = 84$, Heron's Area Formula yields

$$\begin{aligned} \text{Area} &= \sqrt{s(s-a)(s-b)(s-c)} \\ &= \sqrt{84(41)(31)(12)} \\ &\approx 1131.89 \text{ square meters.} \end{aligned}$$

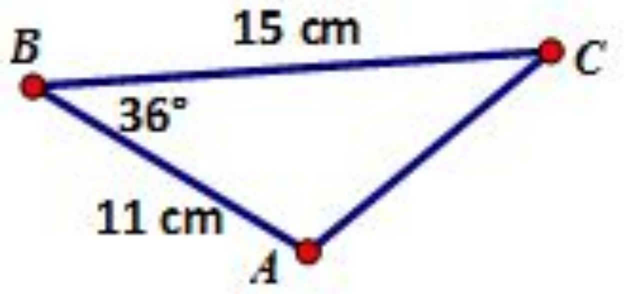
Practice

On a separate piece of paper, determine the area of each of the following triangles. Answers will be provided on the next page.

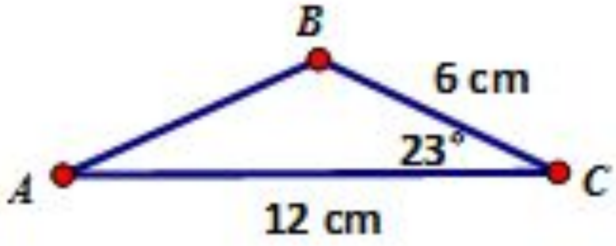
a)



c)



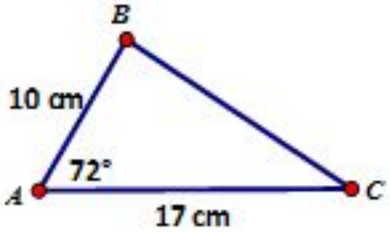
b)



Practice - ANSWERS

On a separate piece of paper, determine the area of each of the following triangles. Answers will be provided on the next page.

a)

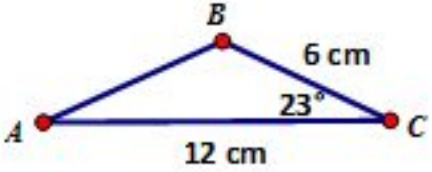


$$Area = \frac{1}{2}bc(Sin\,A)$$

$$Area = \frac{1}{2}(17)(10)(Sin\,72°)$$

$$Area = 80.84\,cm^2$$

b)

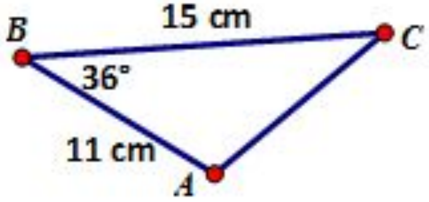


$$Area = \frac{1}{2}ab(Sin\,C)$$

$$Area = \frac{1}{2}(6)(12)(Sin\,23°)$$

$$Area = 14.07\,cm^2$$

c)



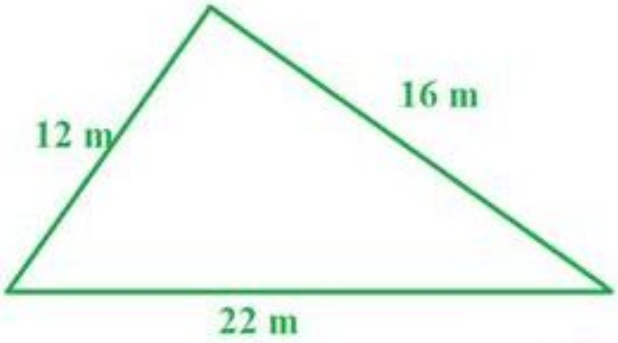
$$Area = \frac{1}{2}ac(Sin\,B)$$

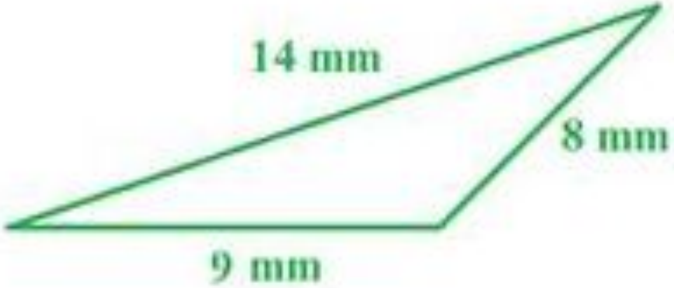
$$Area = \frac{1}{2}(15)(11)(Sin\,36°)$$

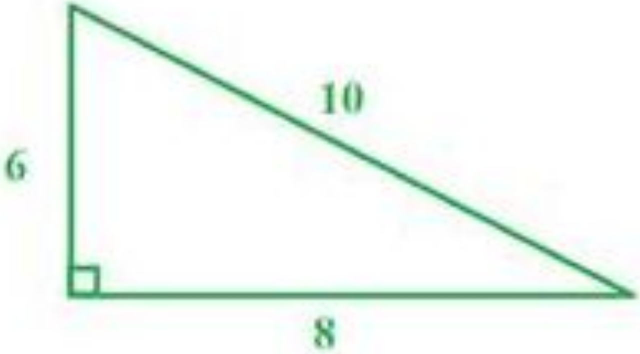
$$Area = 48.49\,cm^2$$

Practice 2

On a separate piece of paper, determine the area of each of the following triangles. Answers will be provided on the next page.







Practice 2 - **ANSWERS**

On a separate piece of paper, determine the area of each of the following triangles. Answers will be provided on the next page.

Step 1: The perimeter of the triangle is equal to 12 + 22 + 16 = 50

Therefore, the s value is half of 50 or 25.

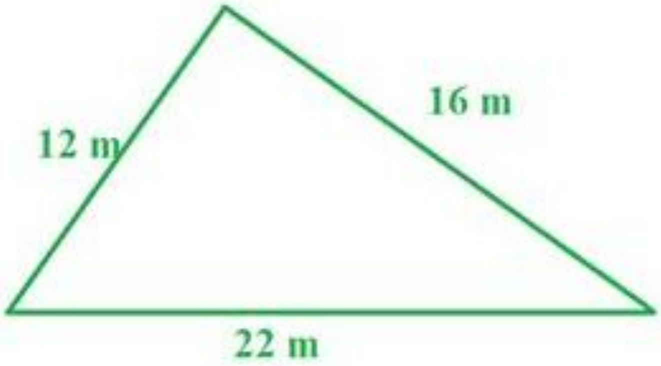
Step 2: Replace the s in the area formula with 25 and solve.

$$A = \sqrt{25(25-12)(25-22)(25-16)}$$

$$A = \sqrt{25(13)(3)(9)}$$

$$A = \sqrt{8775}$$

$$A = 93.7\ m^2$$

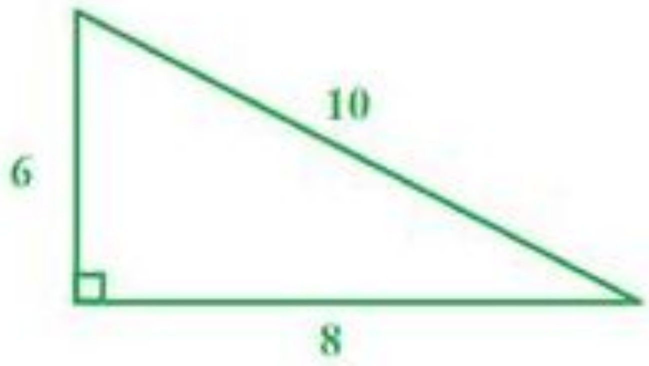


Practice 2 - ANSWERS

On a separate piece of paper, determine the area of each of the following triangles. Answers will be provided on the next page.

Step 1: Determine half the perimeter.

$$s = \frac{6+8+10}{2} = \frac{24}{2} = 12$$



Step 2: Use the s in Heron's formula.

$$A = \sqrt{12(12-6)(12-8)(12-10)}$$

$$A = \sqrt{12(6)(4)(2)}$$

$$A = \sqrt{576}$$

$$A = 24 \text{ units}^2$$

Note: that because this is a right triangle we could have used A = ½bh to get the same answer.

$$A = \frac{bh}{2}$$

$$A = \frac{(6)(8)}{2}$$

$$A = 24 \text{ units}^2$$

Practice 2 - ANSWERS

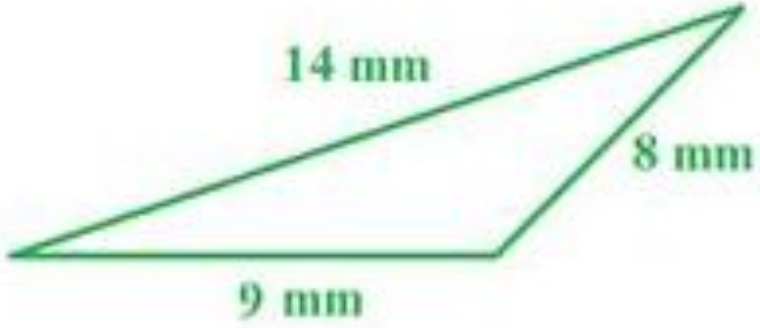
On a separate piece of paper, determine the area of each of the following triangles. Answers will be provided on the next page.

Step1: Determine half the perimeter.

$$s = \frac{9+14+8}{2}$$

$$s = \frac{31}{2}$$

$$s = 15.5$$



Step 2: Use the s in Heron's formula.

$$A = \sqrt{15.5(15.5-9)(15.5-14)(15.5-8)}$$

$$A = \sqrt{15.5(6.5)(1.5)(7.5)}$$

$$A = \sqrt{1133.4375}$$

$$A = 33.7\ mm^2$$

Additional Resource Videos:

[Find the area of an oblique triangle using a formula]

[Area of an Oblique Triangle - SAS & SSS - Heron's Formula, Trigonometry]

Additional Practice:

[Heron's Formula and Law of Sine, Cosine]

[Heron's Formula - Kuta]